SOME APPLICATIONS OF A PRIORI AND A POSTERIORI ERROR ESTIMATES FOR FEM SOLUTION OF NAVIER-STOKES EQUATIONS

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In the application of the FEM in fluid dynamics there are still many open and not well handled problems. One of them is reliable modelling of flows in channels or tubes with abrupt changes of the diameter, which appear often in engineering practice. These are close to the well-known L-shaped domain problems. The goal of this work is to construct the FEM solution in the vicinity of these corners as precise as desired. We present two ways for getting desired precision of the FEM solution near the corners. Both make use of qualitative properties of the mathematical model of flow: the Navier-Stokes equations (NSE) for incompressible fluids.

The first approach makes use of *a posteriori error estimates* of the FEM solution which is carefully derived to trace the quality of the solution. Especially the constant in the a posteriori estimate is investigated with care. Then we use the adaptive strategy to improve the mesh and thus to improve the FEM solution. This method can be quite time demanding, since it needs several runs of solution.

The second approach stands on two stones. One is the asymptotic behaviour of the exact solution of the NSE in the vicinity of the corner. This is obtained using some symmetry of the principal part of the Stokes equation, and application of the Fourier transform. Second stone is the *a priori error estimate* of the FEM solution where we estimate the seminorm of the exact solution by means of the above obtained asymptotics. These ideas allow to derive an algorithm for designing the FEM mesh in advance (a priori). For more details see [1]. We generate the computational mesh in the purpose of uniform distribution of error on elements. On the mesh we then obtain the solution with desired precision also in the vicinity of the corner though there is a singularity here. It gives very precise solution in a cheap way.

Numerical results are presented for flows in channels with either sharp extension or sharp restriction.

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References

[1] Burda, P., Novotný, J., Šístek, J., Precise FEM solution of corner singularity using adjusted mesh applied to axisymmetric and plane flow, Int. J. Num. Meth. Fluids, **47**, 2005, pp. 1285 - 1292.