What is the smallest possible constant in Céa's lemma?

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Abstract

We consider finite element approximations of a second order elliptic problem on a bounded polytopic domain in R^d with $d \in \{1, 2, 3, \ldots\}$. The constant $C \geq 1$ appearing in Céa's lemma and coming from its standard proof can be very large when coefficients of an elliptic operator attain considerably different values. We restrict ourselves to regular families of uniform partitions and linear simplicial elements. Using a lower bound of the interpolation error and the supercloseness between the finite element solution and Lagrange interpolant of the exact solution, we prove that the constant in Céa's lemma can be reduced to $1 + O(h)$ as the discretization parameter h tends to zero. Numerical results in one and two-dimensional case illustrating this reduction are presented.