

Isoparametric quadratic Lagrange finite elements in dimensions one and two

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We characterize all points $a < b < c$ which are nodes of an isoparametric quadratic Lagrange finite element \mathcal{L} in 1D and formulate optimal-order estimates of the L_2 -, H^1 - and H^2 - norms of interpolation errors of functions from the space $H^3(a, c)$ in the nodes a, b, c by interpolants from the local function space of \mathcal{L} . We show that the orders of these norms depend on the way in which the mutual positions of the nodes a, b, c change as the length of interval $[a, c]$ approaches zero.

We study basic properties of the family of triangular isoparametric quadratic Lagrange finite elements \mathcal{K} in 2D with a reference quadratic Lagrange finite element related to a unit triangle \hat{K} with nodes in its vertices and in the midpoints of its sides. We first present general geometric conditions guaranteeing injectivity of a given isoparametric map from \hat{K} and consider some interesting special cases of them including the case in which the images of the midpoints of two sides of \hat{K} are midpoints of the sides of the image of \hat{K} . These conditions, originating from *Jordan, W.B., A. E. C. Research and Development Report KAPL-M-7112, 1970*, can be found in many textbooks devoted to the finite element methods. Then, under the assumption that our general geometric conditions are valid, we formulate the estimates of L_2 -, H^1 - and H^2 - norms of the interpolation error for interpolants from the local function space of \mathcal{K} . The orders of these error-estimates are the same as the orders of the corresponding error-estimates in 1D.

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