The use of basic iterative methods for bounding a solution of a system of linear equations with an M-matrix and positive right side

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A b s t r a c t: In practical applications we handle systems of equations with their matrices being M-matrices and their right sides positive, e.g. in numerical solutions of Markov chains. Especially in its applications on railway safety systems modelling there appear matrices, which possess extremely bad spectral properties. As a consequence the system of equations is badly conditioned, iterative methods converge very slowly and they need more than usual 15 digits of double precision arithmetics. Thus it may be sufficient just to bound the solution (in Markov chains modelling the result means a time before reaching some dangerous state of given system). It may be also useful to ensure that the solution is correct.

Proposed method uses some properties of M-matrices. When we use the basic iterative methods (Ritz, Jacobi and Gauss-Seidel methods), their iteration matrices are nonnegative. We start with an arbitrary vector $\vec{v}$ (when possible, we take a k-th iteration $x^{(k)}$ as $\vec{v}$). We shall then modify this vector $\vec{v}$ with a vector $\vec{\epsilon}$ multiplied with a coefficients $\underline{\delta}$, respectively $\overline{\delta}$, to obtain lower, respectively upper, bounds for solution of given system. These coefficients $\underline{\delta}$,$\overline{\delta}$ are obtained by comparing residual vectors of given $\vec{v}$ and $\vec{\epsilon}$ and distance of these bounds is also affected by residual vectors.

When the residual vector $b - A\vec{v}$ is sufficiently small (compared with right side b of given system), then the constructing of these bounds needs one matrix-vector multiplication and additional O(n) arithmetical operations.