THE DISCONTINUOUS GALERKIN METHOD FOR LOW-MACH FLOWS *

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This work is concerned with the numerical solution of inviscid compressible flow described by the system of the Euler equations using the discontinuous Galerkin finite element method. Our goal is to work out a numerical scheme applicable to flows with a wide range of Mach numbers. In the case of high Mach numbers it is necessary to resolve accurately shock waves, contact discontinuities and (in viscous flow) boundary layers, wakes and their interaction. However, it appears that the solution of low Mach number flow is also rather difficult. This is caused by the stiff behaviour of numerical schemes and acoustic phenomena appearing in low Mach number flows at incompressible limit. In this case, standard finite volume schemes fail. This led to the development of special finite volume techniques allowing the simulation of compressible flow at incompressible limit, which is based on modifications of the Euler or Navier-Stokes equations.

Our goal is to develop a numerical technique allowing the solution of compressible flow with a wide range of the Mach number without any modification of the governing equations. This technique is based on the *discontinuous Galerkin finite element method* (DGFEM), which can be considered as a generalization of the finite volume as well as finite element methods, using advantages of both these techniques. It employs piecewise polynomial approximations without any requirement on the continuity on interfaces between neighbouring elements. In our work we combine piecewise quadratic polynomials in space with a linearized semi-implicit discretization in time which has very good stability properties. In this way we obtain a numerical scheme requiring the solution of only one linear system on each time level. In the low Mach number case a special treatment of boundary conditions is required such that the inlet and outlet are transparent for acoustic phenomena. For transonic flows we apply shock capturing terms, which add artificial diffusion and penalize inter-element jumps near discontinuities.

Computational results show that the presented method is applicable to the numerical solution of inviscid compressible flow ranging from very low Mach numbers (10^{-6}) to transonic flows.

*This work is a part of the research project No. 6/2005/R of the Grant Agency of the Charles University.