On the Role of Reference Maps in the *hp*-FEM

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In this paper we are concerned with the affine concept of the *hp*-FEM, where basis functions in the physical domain are constructed using a set of shape functions defined on a suitable reference domain and appropriate reference maps. Typically, the reference maps depend on the enumeration of vertices in the mesh and/or on other factors. We show that the choice of reference maps may influence the condition number of the stiffness matrix dramatically. Numerical examples are presented, illustrating how much various sets of popular higher-order shape functions depend on reference maps.

In order to eliminate the sensitivity of the computation on the reference maps, we propose to abandon the affine concept and construct basis functions directly in mesh elements. This procedure is elementwise local and therefore easily parallelizable. A new class of higher-order shape functions based on generalized eigenfunctions of the discretized operator is introduced. We show that by abandoning the affine concept, one can improve the conditioning of the discrete problems far beyond the limitations of the affine concept.

References

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